CHAPTER 1

HIGH PRESSURE TECHNIQUES IN GENERAL

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FOREWORD

It is hardly to be thought of that the library of a laboratory which specializes in high pressures, would not contain books on this subject like those of BRIDGMAN [1949], BROWNELL and YOUNG [1959], BUNDY, HIBBARD and STRONG [1960], COMMINGS [1956], HAMANN [1957], MANNING [1963], NEWITT [1940], TONGUE [1959], WENTORF [1962], etc. It is likely, that this library would also contain original texts, published by experts in this field of the science. Consequently an abundant and varied literature about the technology of the high pressures is already made available.

Under these conditions, it seemed to us preferable, to briefly and logically describe the apparatuses used, so that sufficient room can be left for studying the thick-walled cylinder under pressure. Such a study has made considerable progress in the course of the last ten years and with a view to bringing it to a successful issue, we deemed it advisable, to collect many of the available materials, which are still scattered in the technical literature. These materials were dealt with in such a way, that we were able to improve an earlier theory in use at the "Institut Belge des Hautes Pressions" for the design of various apparatuses.

The following questions will be discussed in this chapter. Section 1 gives a general description of high pressure apparatuses and deals with following questions : pressure generators interconnexion of the apparatuses and tight joints.

Section 2 deals with manometric devices. One has enumerated most manometers of standardized types, it being understood, that the pressure balances (or free piston gauges) are included. Theory and practice contribute to considerably improving these pressure balances but we did not dwell upon this subject because it is a subject metrologists are most interested in. We think however of publishing elsewhere a complete technical report about the pressure balances.

Sections 3, 4 and 5 are to be considered as a whole, in which theory and practice simultaneously bring an important contribution to the study of a thick-walled cylinder submitted to a pressure. In section 3, we have tried to throw the light of the recent attainments on the old Lamé's theory. Maxwell's hypothesis has been explained in section 4 by means of notions already known, which brings out such a hypothesis in some relief. In section 5, one has explained an elementary theory of the autofrettage process, which is perfectly utilizable although the scope of its applications is limited. Manning's theory has been then commented without entering into all the particulars of this theory, which is apparently the sole one capable of actually predetermining the bursting pressure of a cylindrical vessel.

In sections 6 and 7 the attention of the reader has been drawn to phenomena of which the theory is very complicated and questionable or simply inexistent, which phenomena in practice must be attached much importance to. The concentration of stresses at the ends of the cylindrical wall, fatigue and corrosion are briefly dealt with in section 6 and in section 7, the temperature effects, that is to say, mainly the creep at high temperatures and the ductility at low temperatures.

We have pleasure in expressing our heart-felt thanks to the "Institut pour l'Encouragement de la Recherche Scientifique dans l'Industrie et l'Agriculture" (I.R.S.I.A.), without the generous contributions of which the Institut Belge des Hautes Pressions would not have been able to bring the study of this important problem to a successful issue.

1. The High Pressure Equipment in General

Any pressure, however high it may be, is normally engendered by using a piston sliding in a cylinder. A thrust applied at one end of the piston is changed at the other end into a hydrostatic pressure within a fluid and even within a ductile solid. Such a thrust is proportional to the pressure engendered and the area of the piston. The hand-operated hydraulic pumps engender a moderate pressure by means of a lever. When said lever is hand-operated, the pressure is increased, when one moves the piston by means of a screw, the nut of which rotates without advancing. Such a device is called a "screw injector". With a view to engendering a pressure, which considerably exceeds above-mentioned one, the piston-cylinder assembly must be placed between the plates of a press. This result may also be achieved by making two pistons of a different section integral one with another. The piston of a big diameter, when moved by a fluid at a low pressure transmits a considerable thrust to the piston of a small diameter, whereas the low pressure fluid is changed into a high pressure one. This device is called a "pressure intensifier".

A piston-cylinder assembly may be made tight by placing a packing either at the end of the piston or at the cylinder inlet. If the first solution is chosen for solving this problem, the bore of the cylinder must be ground. The packing rather quickly wears down but high pressures can be reached. When a screw injector works up to a pressure of 3 kb, we always place a packing at the inlet of the cylinder. By so doing, the piston becomes a simple volume reducer and the cylinder needs not to be ground. Figs. 1a-c schematically show a Bridgman's packing a Poulter's packing and an O-ring seal mounted at the end of the piston. Figs. 1d and e show an Amagat's fully enclosed packing and an O-ring seal, placed at the cylinder inlet.

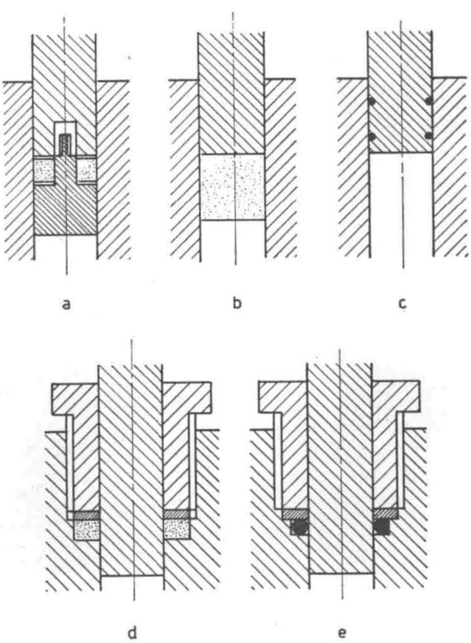


Fig. 1.

A packing mounted at the end of the piston ceases to work correctly, when the pressure exceeds a value of say 30 kb. Pressures above this level can only be engendered by applying the crushing process. BRIDGMAN [1941] enclosed liquids in capsules of lead or of indium. The capsule put into the cylinder was crushed by the piston. The lead is sufficiently plastic for developing a pressure in the liquid and sufficiently viscous for not leaking past the piston. The pressure reached 50 kb. DAVID and HAMANN [1956] applied the crushing process to a polyethylene capsule, closed by a plug made of "teflon". By so doing, they could submit to a pressure of 45 kb a liquid as fluid as methanol.

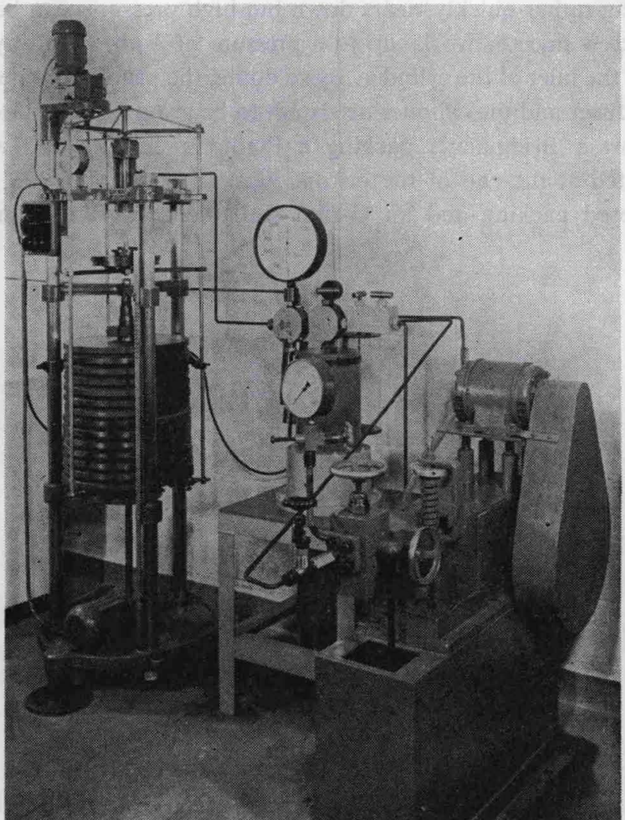


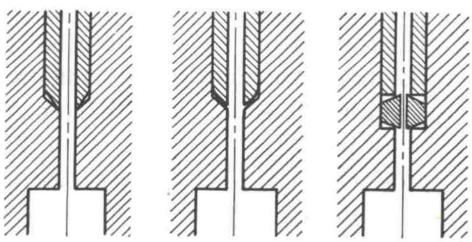
Fig. 1bis.

For the time being, the crushing process is usually applied to small solid samples, which are to be simultaneously submitted to pressures amounting to several hundred kb and to very high temperatures. These techniques are however too specialized ones for being dealt with here.

In our laboratories, pressures between 5 and 10 kb may be developed in the top chamber of the cylinder, in a liquid medium by directly and gradually thrusting the piston upwards into the cylinder either by means of a motor or by means of a piston of a bigger diameter, thrusted itself by a low pressure liquid. The sample, which is to be tested, is placed in the top chamber.

So long as the working pressure does not exceed 10 kb, it is advantageous to generate this pressure in a separate generator and to convey the fluid under pressure into vessels by means of pipes. The generator consists in an oil pump and a pressure intensifier. Owing to the fact that the latter's working is rather fierce, it is usually made use of a screw injector for fine adjustment (fig. 1bis).

A pipe can be connected to the generator or to the vessel by simply crushing steel on steel so that a very small ring-shaped surface of contact acting as a seal is given rise to. The steel is crushed because it has been tightened and the seal will remain tight, so long as the pressure does not exceed a limit which is determined by the tightening forces. This kind of connection is usually used with us and gives satisfaction up to 5 kb. It is obvious, that the hardness numbers of the pieces to be assembled must show not negligible differences.



a b c

Fig. 2.

Figs. 2a–c show some executions of such a junction, the tightening means being omitted. The joint called "cone on cone" and shown by fig. 2a is a joint, which can most easily be executed by using hard steel grades which are very suitable to this purpose. If the pipe is made of stainless, steel, it may happen that it narrows by pinching. The pinching effect however, disappears when the pipe's end is rounded off (fig. 2b) or a lens-ring joint is inserted (fig. 2c).

In more difficult cases, self-sealing packings as the O-ring shown on fig. 3a, or the Bridgman's type of packing shown on fig. 3b, can always be used. In that case, the higher the pressure the tighter the seal. The O-ring, shown on fig. 3a and usually made of rubber can also be made of a metal particularly when it is used for working at a low or a high temperature. Its shape, usually circular, may be replaced by a triangular one so that said seal may be put into a housing, the shape of which is also a triangular one

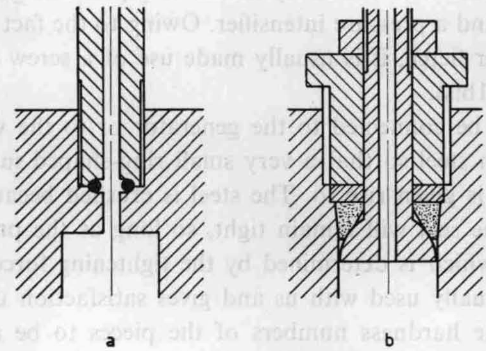


Fig. 3.

Such a seal is called a delta-ring. The wave-ring devised by MANNING [1933] may also be used. The great advantage, which can be taken of using it is that the assembly parts are not submitted to an additional axial thrust but only to lateral thrusts which are easily counterbalanced. The sole objection to using it is that it must be very precisely machined. Failing to do so, it would leak at a low pressure. We almost exclusively use the joint shown on fig. 3b when high temperatures are concerned.

One normally works with vessels as large-bored as possible into which one inserts small apparatuses. If electric signals must be detected or observations taken, the vessel must be provided with suitable electrical connections driven through it or with optical windows. Holes should not be made in the very wall of the cylinder and plugs should normally be provided with connections and windows, which are made tight by applying the same principles. The window shown on fig. 4a and the connector shown on fig. 4b have been made tight by means of a conical shape devised by Amagat. The conical block is not much used now, at least as far as windows are concerned, because it runs the risk of cracking, when the pressure is released. In fact, the steel plug returns to its initial shape and severely presses the window which has been driven through it. figs. 4c and 4d show the principle of the unsupported area, as far as a window (fig. 4c) and a connector (fig. 4d) are concerned. It must be noted that a window can be simply stuck and slightly flattened against the plug. The optical windows have been the subject of a scientific article, written by DEFFET [1943]. With a view to taking observations in the visible spectrum, we recommend the "plexiglas" instead of the glass. A window made of this plastic material is easily executed and easily withstands a pressure of 3 kb.

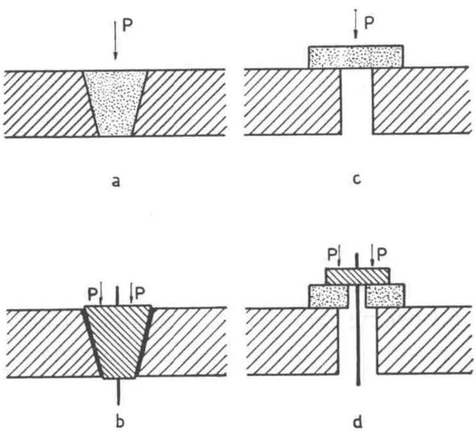


Fig. 4.

The gas compressing technique is so particular, that its brief description should be better given separately. The primary generator may be a shaft or a membrane compressor. The latter delivers a gas, which is kept thoroughly clean; the gas delivered by a shaft compressor and passing through an oil separating cylinder gets sufficiently rid of its oil, so that it can be used for carrying out the usual tests. The gas pressure may be then increased either by means of an oil-mercury-gas separator acting as a mercury pump or by means of an oil-gas separator with mobile piston, acting as a slow moving compressor.

We mention as a reminder the thermal intensifier, which is used in special cases. It simply is a hollow cylinder which is first filled with a gas liquified at a low temperature and then heated until its temperature has reached a higher level.

The devices, which are used for measuring pressures will be enumerated in section 2.

2. The Manometric Devices

The Bourdon gauge with a curved tube is the simpliest and the most frequently used of all the manometers. This type of gauge, if it is of a good make, is reliable but it must be previously calibrated and checked from time to time. It is, however, difficult to get good Bourdon gauges of which the scale exceeds 5 kb. Among the gauges working on the principle of the mechanical deformation, one must still mention the gauge with straight tube and off center hole, which has practically no hysteresis and correctly works up to 10 kb.

The principle of the mechanical deformation or similar principles have been much applied in the course of these last years. The elasticity of a cylindrical or hemispherical wall, submitted to a pressure on one of its faces is made advantage of. The strain of the other face is detected by a sensitive device, generally an electric resistance strain gauge. This sort of pressure gauge is called a "pressure transducer" with a scale remote-reading device. Unfortunately, it requires an expensive amplification. By the way, let us mention that the transducers utilizing the piezo-electric property of the quartz and working by crushing, although they can follow quick variations of the pressure, are not suitable for carrying out static measurements, because the quartz never keeps its electric charge a long time.

The electric resistance gauge with manganine wire, largely used by BRIDGMAN [1949] has been carefully studied by MICHELS and LENSSEN [1934]. The manganine is an alloy, the average composition of which is : copper 80%, manganese 5%, nickel 10% and iron 5%. A manganine wire, coiled and suitably stabilized, has an electric resistance which is pratically a linear function of the hydrostatic pressure, it is submitted to. The specific variation being small, the temperature effects must be eliminated by compensation and a very accurate measuring bridge has to be made use of. Such a manometric coil is suitable for remote-reading very high pressures. Other alloys were tried with less or more success. An alloy, which has been studied by DARLING and NEWHALL [1953] and contains 98% gold and 2% chromium seems to be promising.

The sole manometer of which the systematic errors may be sufficiently well corrected is the mercury column. It is the best standard manometer and actually still the sole manometer of this kind, although great efforts have been made, with a view to making the pressure balance, a standard rival manometer. A mercury column can hardly exceed a height of 25 meters, because the frame-work, supporting it, becomes more and more complicated and because the temperature can no more be said to be uniform along a column exceeding above-mentioned height. By the way, let us mention here the very long columns, once placed by Amagat in a shaft no more used for mining work and by Cailletet in the "Tour Eiffel". The pressures, which may be counterbalanced by a unique column are thus relatively very small. One has tried to extend the scale of the standard manometer either by connecting several columns in series or by using one column submitted at the top to a known counterpressure. By doing so, the correct measuring of pressures is rather complicated.

Therefore, the pressure balance, which is robust, relatively simple and handy is a device, which theoricians and builders take much interest in. By means of a pressure balance, reaching a high level of perfection, it is actually possible, to mesure pressures up to 13 kb.

The essential part of the pressure balance is a piston-cylinder assembly mounted vertically, which piston is pulled down by weights and by the atmospherical pressure and pulled up by the thrust engendered by a compressed fluid. Either the piston in most cases or sometimes the cylinder rotates or oscillates with a view to reducing to almost nil the friction metal on metal. The dry friction is practically eliminated, when the piston speed reaches a critical level and this fact has been evidenced by applying a difference of potential between the piston and the cylinder. A constant pressure being maintained under the piston, the fluid slowly flows along the piston, which in its turn moves down. The fluid leakage simultaneously increases with the pressure, because of the progressive distortion of the parts, unless the clearance is controlled by a counterpressure, applied outside the cylinder, so that the leakage may be practically reduced to nil. The viscous friction causes the piston to move upwards. It may be asked whether an accessory thrust is caused by the rotation of a piston, which bears helicoïdal traces of tools or not. This thrust can be evidenced by reversing the direction of rotation. From this brief description we gather, that a pressure balance is a piston-cylinder-fluid assembly, the correct working of which is on one side, but not necessarily, limited by the leakage of the fluid and on the other side, and this necessarily, by the viscosity, which can become extraordinarily high and in any way by the mechanical resistance of the piston-cylinder assembly.

The simplest type, which takes up the least amount of space, is the hand-rotated pressure balance with weighty disks, piled up on a plate, surmounting the piston. An other hand-rotated type, which is nearer to perfection comprises a long cap, of which the head rests on the piston and on the lower brim of which holed disks are piled up. But the pressure balances, which are nearest to perfection have an impressing superstructure, which is necessary for keeping either the piston or the cylinder permanently and regularly rotating or oscillating and for charging said balances with weights, which are normally placed lower than the piston and preferably by means of a mechanical device.

There are various types of piston-cylinder assemblies. The simplest is the one shown by fig. 5a, the piston of this assembly being not necessarily terminated by a stop. fig. 5b shows a differential piston, the annular shoulder

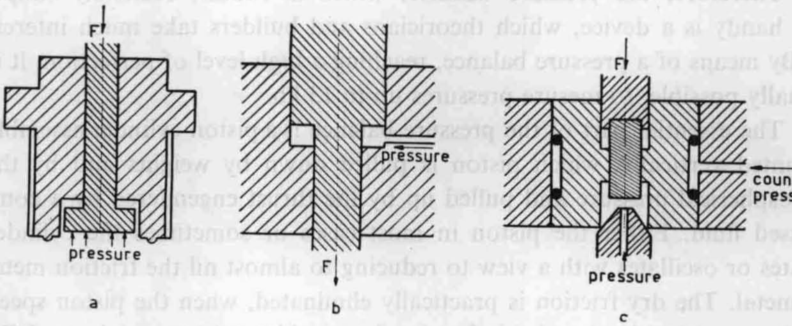


Fig. 5.

of which is submitted to a thrust, which decreases accordingly, as the shoulder becomes narrower. The pressure balance with differential piston has been particularly developed at the National Physical Laboratory (Teddington) and by Michels at the "Van der Waals Laboratorium" (Amsterdam). This pressure balance can correctly measure pressure up to 3 kb. The assembly piston-cylinder with controlled clearance, shown at fig. 5c is constructed in the USA. The normal model, measuring pressures up to 7 kb is very handy. Provided that an appropriate fluid may be found, a pressure balance of that type is capable of measuring pressures up to 13 kb. An interesting study, which we refer the reader to and which deals with this subject has been published by Johnson and Newhall [1957]. Fig. 5c shows how it is possible to adjust the clearance to a suitable value, by applying outside the cylinder a counterpressure proportionate to the pressure which is to be measured.

Precision machine tools can now be used for machining a piston, which is sufficiently cylindrical so that, assuming that the piston of a balance would remain rigid and work without being affected by the friction of the fluid, the value of the pressure prevailing under the piston can be obtained by the formula $p = (F/A_0) + P_{\text{atm}}$, in which F denotes the total of the weights acting on the piston, A_0 the latter's area and P_{atm} the atmospheric pressure at a given moment; thrust resulting from the application of Archimedes' principle have been discarded here, as being incorporated in trivial corrections. For simplicity's sake and with a view to taking frictions and distortion into account, one continues to represent the pressure by the simple formula $p = (F/A_e) + P_{atm}$, in which the symbol A_e is put instead of symbol A_0, A_e being the "effective area" of the piston. All the difficulties of the problem are concentrated in the symbol A_e, which is a function of the pressure still to be determined and considerable efforts are made by theoricians and experimenters with a view to improving their knowledge of this function.

The theory of the pressure balance has been built up by DEFFET and TRAPPENIERS [1954], who to this purpose made use of a bibliography going up to the year 1950. A pressure balance based on this theory and measuring pressures up to 3 kb (see fig. 1bis) has been constructed by the "Compagnie des Compteurs et Manomètres, Liège". But since that time, the situation has improved.

The experimental determination of A_e has been made by using two methods. NEWITT and his collaborators developed a method, in which the pressure balance is compared to a mercury column, submitted to a counter-pressure. The first results of this method described by BETT, HAYES and NEWITT [1954] have been published by BETT and NEWITT [1962] and show, that the effective areas of the piston, when an appropriate fluid is made use of, is practically a linear function of the pressure.

DADSON [1957] developed an other method, which consists in comparing two piston-cylinder assemblies, geometrically quasi identical, but made of different materials. Here too, the results obtained by using this method, show that A_e is a linear function of the pressure. The slope of the straight line representing this function depends upon the fluid utilized. The paraffin oil makes however an exception and causes A_e, to vary regularly but not linearly.

From a report not yet published but kindly communicated to us by R.S. DADSON [1963], we extract further valuable information. Dadson's method has been extended up to 7 kb. and successfully tested by means of a third material of comparison. The distortion coefficients of the pressure balances are actually known with an accuracy reaching 2% or 3%. The effective areas are known with an overall accuracy estimated at 1 part in 10^5 at low pressure and at about 8 parts in 10^5 at the upper end of the range. Dadson's method has been also successfully tested in a fully independent way by comparing the leakage rates of flow. On the other side air-operated pressure balances made it possible recently to measure pressures well below 1 b by weighing them and by so doing, to measure a range of pressures, which historically were measured by making use of the mercury column. The measures made by means of columns and by means of pressure balances are in agreement to the order of 1 part in 10^5, as far as their common working interval is concerned.

The question would be incompletely dealt with in this section if we would not say anything about certain charactcristic pressures, which in the pressures measuring technique would play the same role as the fixed points in thermometry, when these characteristic pressures will be known with the desirable acuracy. The vapour pressure of the carbon dioxide at the water freezing point is the best known of above-mentionned characteristic pressures. MICHELS *et al.* [1950] found that it was equal to 34.391 atm which corresponds to 34.400 atm at the temperature of the water triple point; values found by other experimenters are quoted in this article. BRIDGMAN [1949] pointed out other interesting characteristic pressures. The mercury freezes at the water freezing point under a pressure of 7 640 kg cm^{-2} but its temperature coefficient is very high:

$$(\partial p/\partial T)_v = 210 \text{ kg cm}^{-2} \text{ deg}^{-1}.$$

The bismuth I–II transition occurs under a pressure amounting to 25 200 kg cm^{-2} approximately. The different water triple points are connected with following characteristic pressures:

$$(l - \text{I–III}) \approx 2\,000 \text{ kg cm}^{-2},\ (l - \text{III–V}) \approx 3\,500 \text{ kg cm}^{-2},$$
$$(l - \text{V–VI}) \approx 6\,300 \text{ kg cm}^{-2},\ (l - \text{VI–VII}) \approx 22\,000 \text{ kg cm}^{-2}.$$

R.S. DADSON [1963] mentioned in his private communication, that the value found at the National Physical Laboratory (Teddington) for the mercury freezing pressure at a temperature of 0°C is equal to 7 720 kg cm^{-2}, and exceeds thus by 1% the value published by Bridgman; on the other hand, the value found for the vapour pressure of the carbon dioxide is nearly equal to this one found by Michels.

3. The Elastic Equilibrium of a Thick-Walled Cylinder

Les us consider a cylindrical wall submitted to an internal pressure p_1 and an external one p_2. The outer and inner radii are indicated by r_1 and $r_2 = kr_1$; k is the diameter ratio. One intermediate radius is preferably indicated by lr_1; l varies from 1 to k, when r varies from r_1 to r_2. σ_t being the tangential stress, the equilibrium of the forces, shown by fig. 6a obeys the following relation

$$2\,p_2 k r_1 + 2\int_1^k \sigma_t\,\mathrm{d}\,(l r_1) = 2\,p_1 r_1$$

or

$$\int_1^k \sigma_t\,\mathrm{d}l = -\int_1^k \mathrm{d}\,(pl) = \int_1^k \mathrm{d}\,(\sigma_r l). \qquad (1)$$

In the last integral, the pressure p, considered as a positive quantity, has been replaced by $-\sigma_r$ according to the sign convention. σ_r is the radial stress.

Eq. (1) being true whichever value k may have, one can write

$$\sigma_t\,\mathrm{d}l = \mathrm{d}\,(\sigma_r l) \quad \text{or} \quad \sigma_t - \sigma_r = l\,\frac{\mathrm{d}\sigma_r}{\mathrm{d}l}. \qquad (2)$$

Eq. (2) is true for the elastic as for the plastic state as far as the deformed wall remains perfectly straight, circular and concentric. If however the deformation of the wall becomes important one must put down $r + u$ into eq. (2) instead of r, because the equilibrium of the forces refers to the deformed wall and not to the shape, the wall has, when it is at rest.

Only considering now the very small elastic deformations, one can write the following relations expressing the tangential strain ε_t and the radial strain ε_r

$$\varepsilon_t = \frac{2\pi\,(r+u) - 2\pi r}{2\pi r} = \frac{u}{r},$$

$$\varepsilon_r = \frac{[(r + \mathrm{d}r + u + \mathrm{d}u) - (r+u)] - \mathrm{d}r}{\mathrm{d}r} = \frac{\mathrm{d}u}{\mathrm{d}r}.$$

ε_t and ε_r are not independent, because after deriving ε_t and eliminating $\mathrm{d}u/\mathrm{d}r$, one finds eq. (3), which is a consequence of the cylindrical symmetry and is called the "comptability equation"

$$\varepsilon_r - \quad = r\,\frac{\mathrm{d}\varepsilon_t}{\mathrm{d}r} = l\,\frac{\mathrm{d}\varepsilon_t}{\mathrm{d}l}. \qquad (3)$$

As a rule, it is sufficient to express eq. (3) in terms of the stresses σ_t and σ_r and of the axial stress σ_z and the problem may be now considered as solved, because we dispose of eqs. (2) and (3) and also of an equation, which is not yet written but is the expression, which the forces axial eqiulibrium obeys. For simplifying this problem, one must assume as LAME [1852] did, that the transverse sections of the wall remain plane after deformation.

This assumption has been experimentally confirmed provided these sections are sufficiently remote from the ends of the cylinder considered. A wall, which is submitted to pressures does neither bend nor show any other distorsion than an axial and symetric deformation. We shall consequently put down ε_z = constant or $d\varepsilon_z/dl = 0$.

One can write now following relations, based on Hooke's law and in which the strains are linear functions of the stresses

$$E\,\varepsilon_t = \sigma_t - \nu\,(\sigma_r + \sigma_z), \tag{4a}$$

$$E\,\varepsilon_r = \sigma_r - \nu\,(\sigma_t + \sigma_z), \tag{4b}$$

$$E\,\varepsilon_z = \sigma_z - \nu\,(\sigma_t + \sigma_r), \tag{4c}$$

E and ν being the Young's modulus and Poisson's ratio.

By making use of eqs. (4a-c) one forms the expressions $\varepsilon_r - \varepsilon_t$ and $\varepsilon_t + \nu\,\varepsilon_z$. which are introduced into eq. (3), the right hand side of which can also be written $d\,(\varepsilon_t + \nu\,\varepsilon_z)/dl$ because ε_z is constant. By doing so, eq. (3) is expressed in terms of the stresses

$$\sigma_t - \sigma_r = \nu l\,\frac{d\sigma_r}{dl} - (1-\nu)\,l\,\frac{d\sigma_t}{dl}. \tag{5}$$

Eqs. (5) and (2) give each an expression of $\sigma_t - \sigma_r$. One can write by comparing these expressions together : $d\,(\sigma_t + \sigma_r)/dl = 0$, and consequently $\sigma_t + \sigma_r = 2A$. By replacing σ_t by $2A - \sigma_r$ in eq. (2) one finds the value of σ_r after integration

$$\sigma_r = A - Bl^{-2} \tag{6}$$

and also the value of σ_t

$$\sigma_t = A + Bl^{-2}, \tag{7}$$

A and B being two constants still undetermined. On the other side, eq. (4c) shows now, that σ_z is constant

$$\sigma_z = C. \tag{8}$$

This is an interesting result indeed, because the stresses themselves cannot be measured by making adequate experiments to that purpose.

The value of the constants A, B and C, depend upon the case envisaged. As it has been decided, to submit the wall to an internal pressure and an external one, one must put down into eq. (6) $\sigma_r = -p_1$ when $l = 1$ and $\sigma_r = -p_2$ when $l = k$, so that two particular relations can be made available and made use of, for easily determining the values of constants A and B, which introduced into eqs. (6) and (7) change the latter into eqs. (9) and (10) of table 1. Constant C depends upon how the wall is obturated. For instance fig. 6b represents a wall, closed by plugs, which transmit to the latter an axial thrust. The axial forces equilibrium, which has yet not been envisaged leads to the following equation :

$$\pi p_2 (kr_1)^2 + \pi \sigma_z (k^2 r_1^2 - r_1^2) = \pi p_1 r_1^2$$

and consequently to eq. (12) of table 1. Fig. 6c shows a wall, closed by plugs and only submitted to an internal pressure. This case, which often occur in the engineering practice, is described by eqs. (9) to (12) of table1, provided

TABLE 1

$$\sigma_r = \frac{p_1 - k^2 p_2}{k^2 - 1} - \frac{p_1 - p_2}{k^2 - 1} \frac{k^2}{l^2} \tag{9}$$

$$\sigma_t = \frac{p_1 - k^2 p_2}{k^2 - 1} + \frac{p_1 - p_2}{k^2 - 1} \frac{k^2}{l^2} \tag{10}$$

$$\tau = \frac{p_1 - p_2}{k^2 - 1} \frac{k^2}{l^2} \tag{11}$$

$$\sigma_z = \frac{p_1 - k^2 p_2}{k^2 - 1} \tag{12}$$

$$Eu = \left[(1 - 2\nu) \frac{p_1 - k^2 p_2}{k^2 - 1} + (1 + \nu) \frac{p_1 - p_2}{k^2 - 1} \frac{k^2}{l_2} \right] lr_1 \tag{13}$$

that p_2 is equal to zero in these equations. Finally, fig. 6d shows a wall, which has not to withstand an appreciable axial thrust ($\sigma_z = 0$), because on one side this thrust is transmitted to a rigid support and on the other side to a counterbalanced piston. This is the case of a press and a pressure balance. The same equation $\sigma_z = 0$ applies to the more exceptional case of a wall, closed on both sides by counterbalanced pistons, provided that friction effects between the wall and the piston packing are neglected. Table 1 has been completed by eq. (11), giving the value of the shear stress $\tau = \frac{1}{2}(\sigma_t - \sigma_r)$ and by eq. (13) giving the analytical expression of the displacement of radius lr_1. Considering that $\varepsilon_t = u/r = u/lr_1$ and making use of eqs. (4a), (9), (10) and (12), one easily finds eq. (13). This last equation is an interesting one, because the displacement u_2 of the outer radius ($l = k$) can be easily measured by means of an extensometer, when the external pressure is nil. After carrying out some measurements under growing pressures p_1, one can extract from eq. (13), the value of E and this one of ν pertaining to the material in question and such values can be said to be fully as good as values obtained by other methods.

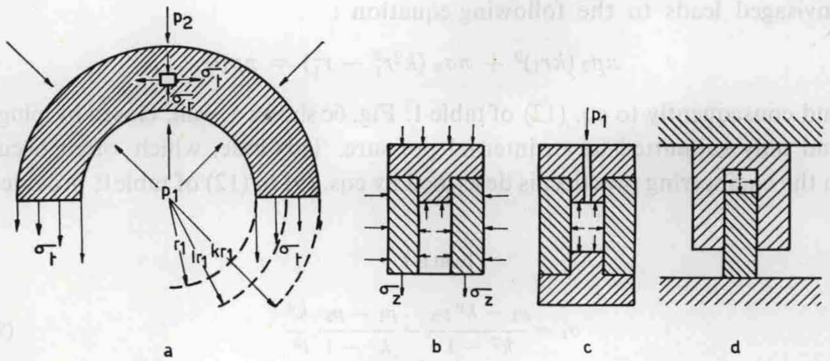


Fig. 6.

It is essential to carefully examine the eqs. (9) to (12) :

(a) The stresses result from the superposition of a stress of a hydrostatic nature equal to σ_z and of a pure shear stress τ, which depends on the sole difference of the applied pressures.

(b) The shear stress τ, which is of prime importance in the wall overstraining process, as shown in section 5, can be diminished by forcing an inner wall into an outer one with a view to increasing the resistance of said inner wall to a pressure applied on its inner surface. In order to achieve this aim, the inner wall is machined so that its outer diameter is a little bit greater than the inner diameter of the outer wall, which will be heated to a temperature much higher than the inner wall's one, before compoun ding. By cooling down the outer wall is pressed against the inner one under a pressure equal to p_2, when the temperatures of both walls have reached the same level. This is the classical compounding by shrinkage, which is an expensive and delicate wall resistance reinforcing process, because the walls must be machined by means of high precision machines and also because it may happen, that in course of carrying out this process, both cylinders stick together so strongly, that it is impossible to fully insert the inner cylinder into the jacket. As MANNING [1947] made it clear, it must be noted that defaults due to faulty workmanship may considerably lower the shrinkage efficiency. The cylinders of pumps and compressors are often reinforced with steel jackets with a view to reducing the risks of fatigue cracks.

Suitable reinforcement processes have been naturally thought of, in the industry. The big chemical reactors are actually built by winding under tension strong bands around a thinwalled cylinder, the material of which is selected for its good resistance to corrosion, although its mechanical resistance is rather low.

BRIDGMAN [1936] has thought out a method for progressively reinforcing the resistance of cylinders, which is applicable to high pressure apparatuses. The cylinder is given a slight outside conicity. It is then progressively forced into another cylinder, which has a slight inside conicity. In the original disposition the thrust resulting from the pressure, to which the inner face of the cylinder head is submitted, progressively presses both cylinders one against another. At the upper cylinder head, the pressure is equilibrated by a piston. In an other disposition, invented by BRIDGMAN [1940] and utilized by DAVID and HAMANN [1956] up to 45 kb two cylinders are used for reinforcing the inner one. The cylinders are progressively pressed one against the other by means of a press.

With a view to crushing solid samples up to 100 kb and more, the pistons and cylinders are reinforced especially in the "Belt" apparatus, designed by HALL [1960]. A description of the Belt apparatus as well as of other apparatuses of various types can be found in a article, written by BUNDY and inserted in the text of a treatise, published by WENTORF [1962].

(c) When the external pressure is practically nil, as it is generally the case, the stresses are such, that following inequalities can be written : $\sigma_t > \sigma_z > \sigma_r$, so that $\tau = \frac{1}{2}(\sigma_t - \sigma_r)$ is "ipso facto" the greatest shear stress.

(d) If walls having different inner radii whereas the k ratio remains the same, are submitted to the same pressures p_1 and p_2, they will present the same stress configuration at places, defined by the same l. This important principle of similarity excludes every scale effect in stress configurations, where the elasticity laws apply, it being however understood, that the walls are made of a perfectly homogenous material, that their geometry is a perfect one and that Hooke's law is applicable to this material.

When a pressure is applied to the inner side only of a cylinder ($p_2 = 0$), when said pressure is mesured by means of a pressure balance and when the outer and even the inner deformation is measured by means of strain gauges, measuring microstrains, it has been found, that the stress-strain curve slightly deviates from the straight line, which is typical of Hooke's law graphical representation, owing to a crystalline microplasticity which appears very soon and develops within a very short time. Now, the stresses at the bore ($l = 1$) are numerically the greatest. Assuming that the macroscopic overstraining depends on the stresses, such an overstraining (plasticity) will first appear at the bore. This phenomenon can easily be observed, because the inside and outside deformations, as far as steel grades with low carbon content are concerned, suddenly develop and because their stabilization takes some time.

The question of determining whether this overstraining is affected by a scale effect is more difficult and delicate to settle out once and for all. If this phenomenon depends on the sole stresses, there is no scale effect, because the principle of similarity is in this case applicable but if it also depends on the shear stress gradient, a scale effect may appear, because the shear stress gradient $\mathrm{d}\tau/\mathrm{d}(lr_1) = (\mathrm{d}\tau/\mathrm{d}l)\, r_1^{-1}$ varies with the inner radius. This problem has been studied by DEFFET and LIALINE [1959] and LIALINE [1959].

4. The Elastic Energy of a Thick-Walled Cylinder

Let us consider a small cube of material, of which the edge is equal to the unit of length and which is oriented, so that the principal stresses σ_1, σ_2 and σ_3 act on its faces. When the edges are increased by $\mathrm{d}\,\varepsilon_1$, $\mathrm{d}\,\varepsilon_2$ and $\mathrm{d}\,\varepsilon_3$, the elastic energy of the cube, namely the energy per unit of volume is increased by $\mathrm{d}W$

$$\mathrm{d}W = \sigma_1 \mathrm{d}\varepsilon_1 + \sigma_2 \mathrm{d}\varepsilon_2 + \sigma_3 \mathrm{d}\varepsilon_3.$$

By applying Hooke's law, which leads to equations, similar to eqs. (4a-c), $\mathrm{d}W$ can be expressed as follows

$$\mathrm{d}W = \frac{1}{2E}\,\mathrm{d}\,(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - \frac{\nu}{E}\,\mathrm{d}\,(\sigma_1\,\sigma_2 + \sigma_2\,\sigma_3 + \sigma_3\,\sigma_1).$$

By superimposing upon the stresses acting on the cube a certain hydrostatic state, characterized by three principal stresses equal to $-\sigma$, one submits the cube to the combined stresses : $s_1 = \sigma_1 - \sigma$, $s_2 = \sigma_2 - \sigma$, $s_3 = \sigma_3 - \sigma$. In this case the energy is increased by $\mathrm{d}W$, expressed by following equation, taking into account that $\sigma_1 = s_1 + \sigma$, $\sigma_2 = s_2 + \sigma$, $\sigma_3 = s_3 + \sigma$

$$\mathrm{d}\,W = \frac{1}{2E}\,\mathrm{d}\,(s_1^2 + s_2^2 + s_3^2) - \frac{\nu}{E}\,\mathrm{d}\,(s_1\,s_2 + s_2\,s_3 + s_3\,s_1)$$

$$+ \frac{3\,(1 - 2\,\nu)}{2E}\,\mathrm{d}\sigma^2 + \frac{1 - 2\,\nu}{E}\,\mathrm{d}\,(s_1 + s_2 + s_3)\,\sigma.$$

It is clear, that the term $(\frac{3}{2}E)\,(1 - 2\,\nu)\,\mathrm{d}\sigma^2$ represents the work of the pressure $-\sigma$, when the volume of the cube changes. The term $[(1-2\nu)/E]\,\mathrm{d}(s_1+s_2+s_3)\,\sigma$, the meaning of which cannot intuitively be caught, can be done away with by putting down : $s_1+s_2+s_3=0$, that is to say by putting down $\sigma=\frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)=\sigma_m$. By making use of following identity : $0=(s_1+s_2+s_3)^2=s_1^2+s_2^2+s_3^2+2(s_1s_2+s_2s_3+s_3s_1)$ one obtains $\mathrm{d}W$'s final expression

$$\mathrm{d}W=\frac{1+\nu}{2E}\,\mathrm{d}(s_1^2+s_2^2+s_3^2)+\frac{3(1-2\nu)}{2E}\,\mathrm{d}\sigma_m^2$$

= distortion energy + volume change energy.

This relation shows, that, in order to obtain the pure distortion energy, the average stress σ_m has to be substracted from stresses σ_1, σ_2 and σ_3 and that the sole residual stresses s_1, s_2 and s_3 are to be taken into account. The finite form of the distortion energy is obtained by integrating $\mathrm{d}W$ from an initial unstressed state up to a final one, characterized by s_1, s_2, s_3. Continuing to call the sole distortion energy by W, one obtains following relation

$$W=\frac{1+\nu}{2E}(s_1^2+s_2^2+s_3^2).$$

In the case of a thick-walled cylinder, one extracts from eqs. (9), (10) and (12) following relations

$$-s_r=s_t=\frac{p_1-p_2}{k^2-1}\,\frac{k^2}{l^2}\quad\text{and}\quad s_z=0$$

so that the specific wall distortion energy is expressed as follows

$$W_l=\frac{1+\nu}{E}\left(\frac{p_1-p_2}{k^2-1}\,\frac{k^2}{l^2}\right)^2.$$

Subindex l has been added to W, in order to draw the attention to the fact, that the energy varies with l. Considering the most frequent case of wall submitted to an internal pressure only ($p_2=0$), the greatest energy is to be found near the bore of the cylinder ($l=1$)

$$W_{l=1}=\frac{1+\nu}{E}\left(\frac{p_1\,k^2}{k^2-1}\right)^2.$$

Considering now a tensile test-piece, which has reached the upper yield stress σ_y ($\sigma_1=\sigma_y$, $\sigma_2=\sigma_3=0$) one finds $s_1=\frac{2}{3}\sigma_y$ and $s_2=s_3$ and a maximum uniformly distributed distortion energy in the test-piece

$$W_y=\{(1+\nu)/E\}\,\tfrac{1}{3}\,\sigma_y^2.$$

As Maxwell did, let us now assume, that all the bodies of a same material cease to have elastic reactions at places where and at the moment when their distortion energy reaches a critical level. The critical level of the cylinder energy $W_{l=1}$ must be then equal to the maximum energy W_{y} of the tensile test-piece, so that the critical pressure $p_{1\mathrm{y}}$ is a function of the upper yield stress σ_{y} and is expressed by following relation

$$p_{1\mathrm{y}} = (1 - 1/k^2)\, \sigma_{\mathrm{y}}/\sqrt{3}. \tag{14}$$

Eq. (14) is applicable, as shown in following paragraph.

5. The Plastic Flow of a Thick-Walled Cylinder

By gradually increasing the pressure, applied inside the cylinder, the material layers at the bore show first an elastic deformation, then a plastic one, which begins, when the pressure has reached a critical level $p_{1\mathrm{y}}$ called the yield pressure. The plastic deformation propagates through the wall, under increasing pressure, until the whole material has become plastic, which happens, when the pressure has reached a level $p_{1\mathrm{c}}$ called the "collapse" pressure. The constant pressure $p_{1\mathrm{c}}$ produces greater deformations in ductile materials, characterized by a small k ratio, until the material layers at the bore begin to harden. Then the pressure must be increased, in order to increase the deformation, because a more and more important part of the material hardens. At the moment when the deformed wall thins down and simultaneously hardens, so that the two phenomenons are compensated, the pressure applied, reaches its peak level. This maximum pressure $p_{1\mathrm{u}}$ is called the ultimate pressure. The wall swells rapidly at a determined place and bursts at a pressure, which is lower than $p_{1\mathrm{u}}$ and is of little interest. The wall remains thus cylindrical, until pressure $p_{1\mathrm{u}}$ is reached.

There is a pressure range, which is particularly interesting to study, and it is this one, comprised between pressures $p_{1\mathrm{y}}$ and $p_{1\mathrm{c}}$, in other words, the range, in which the deformations produced cannot freely develop and as far as their smallness is concerned, are still comparable with elastic deformations. When the pressure is going down, the wall portion, submitted to an elastic deformation shows a tendency to reassume its initial shape but it is hindered from doing so by the underlying wall portion, which has been overstrained. Residual stresses consequently appear, which are to a certain extent similar to these appearing in an elastic wall, reinforced by shrinkage. This is the reason why a wall made of a material, which has been partially submitted to plastic deformations is called "self-hooped" or reinforced by "autofrettage". The idea of self-hooping a wall occurred in 1906 to Malaval, a French engineer of the Naval Artillery Department and apparently was applied for the first time in 1913 to the construction of a high caliber gun. The same idea was resumed by LANGENBERG [1925] then by MACRAE [1930], which devoted much time to studying the problem of stabilizing the self-hooped cylinders. In the field of the high pressures the "autofrettage" process may be easily applied to parts, of which the bore's regularity is of very little importance, provided that this process is applied with appropriate precautions, for instance to pipes and vessels. But its application to a cylinder, in which a shaft moves to and fro is more delicate, because the bore of the cylinder must be ground after applying such a process and because the cylinder in most cases must be stabilized by heat treating it.

In the first place, it is important to know whether the plasticity, which has developed in a cylinder at a yield pressure p_{1y}, will concentrically propagate. If the answer to the question is no, the theoritical considerations, we would develop for explaining the facts, run the risk of being so complicated, that it could no more be hoped, that such facts could be explained by having recourse to a simple theory. By putting very sensitive extensometers in the direction of diameters crossed at a right angle, CROSSLAND and BONES [1958] found, that measurements taken on cylinders, made of a 0.15% carbon steel, showed marked and suddenly appearing differences, which can be attributed to a dissymmetrical progression of the plasticity; but when one plots a diagram by means of the average of the readings, one obtains a regular curve and this is a very encouraging result indeed. When the whole cylinder is overstrained, the readings scattering reduces to nil; thus when the pressure reaches the level p_{1c} the test cylinder shows concentrical deformations. The same authors have observed, that in a low alloy steel, the plastic zone front moves forward concentrically from the beginning to the end of the experience.

COOK [1934] had already noted that mild steel cylinders gave rise to divergent extensometric readings, whereas STEELE and YOUNG [1952] have demonstrated by chemically attacking polished surfaces, that the mild steel plasticity propagates in the form of veins through an ambient material, which remains elastic. As it can be seen, following metals are an exception : the mild steel and probably the materials, which show, when they are submitted to tensile or torsion tests, an upper yield stress and a lower yield stress which are sufficiently remote one from another. The mild steel could obviously be excluded from the theory, which will follow, because it is no more used in the technique of high pressures but it is better to include it in said theory, it being understood, that its deformations will be characterized by the average of the extensometric readings.

Now, we would like to know whether and how the yield pressure p_{1y} may be more directly connected to other critical quantities, derived from tensile, flexional or torsional tests results. The first vague and confused notions about this problem, are now going to solve. CROSSLAND [1954], who most contributed to achieve this aim, showed experimentally, that the fact of superimposing very high hydrostatic pressures affects in any way neither the upper yield shear stress τ_y of materials submitted to torsional tests nor their plastic yielding. Test rods were the subject of very precise tests, which fully succeeded, because the Morisson's joint used by the experimenters and described in the same article rubbed very little against the rod. Most materials tested have however been more strongly deformed at breaking point by the sole effect of the surrounding pressure. From these results it appears that a cylindrical wall, closed at both ends and submitted to an internal pressure must overstrain, when its major shear stress given by eq. (11) reaches the yield level τ_y extracted from the results of a torsional test. One puts thus down into eq. (11) $p_1 = p_{1y}$, $p_2 = 0$; $l = 1$ and $\tau = \tau_y$ so that the yield pressure can be obtained

$$p_{1y} = (1 - 1/k^2)\,\tau_y. \tag{15}$$

As it can be expected, the mild steel and other materials of the same type are unfortunately exceptions. MORRISON [1940] found, that their upper yield shear stress is a linear function of the shear stress gradient, so that τ_y in eq. (15) must be corrected.

Eq. (15), although it is perfectly reliable, is unfortunately a function of the quantity τ_y,which until now is not included in the data the makers of high grade steel supply us with. With a view to determining this quantity τ_y, one must either carry out a torsion test or base the guess on Maxwell's or Tresca's assumptions. As we already mentioned it, Maxwell's assumption leads to eq. (14) and results in identifying τ_y with $\sigma_y/\sqrt{3}$. The other assumption results in identifying τ_y with $\frac{1}{2}\sigma_y$, because Tresca assumed, that a body begins to yield at the place where and at the moment when the major shear stress has reached a critical level and this, wichever the kind of stresses occurring may be. The upper yield shear stress in tension is precisely equal to $\frac{1}{2}\sigma_y$. DEFFET and GELBGRAS [1953] experiments as well as CROSSLAND and BONES' one [1958] have shown, that generally speaking, the results of these experiments can be better accounted for by Maxwell's assumption than by Tresca's one, although they are exaggerated. Tresca's assumption, which leads to underestimated results is nevertheless advantageous, because the safety is much better taken into account. Personally, we have recourse to Tresca's hypothesis, when we design new apparatuses.

Eq. (15) shows, that it is little advantageous to choose a k ratio above 3. The wall elastic resistance is only increased by 5%, when one passes from $k = 3$ to $k = 4$, whereas the weight of the cylinder is increased by 87%.

We will tackle now the problem of an overstrained cylinder. In the first place we will make our readers acquainted with an elementary solution, which can be expressed algebraically and owing to this advantage, completes Lame's formulae. We will then briefly describe a more refined solution elaborated by Manning and necessitating numerical integrations. Both theories, on which above-mentioned solutions are based, assume, that the plastic front obeys the law of the cylindrical symmetry. The elementary solution is based on Cook's hypothesis : the axial stress is equal to the arithmetical mean of the other main stresses : $\sigma_z = \frac{1}{2}(\sigma_t + \sigma_r)$ as well in the elastic zone of the wall as in the plastic zone of it; it is to be noted, that eqs. (9) to (12) of table 1 can be used, when the solution of the problem is based on Cook's hypothesis, although obviously the whole axial load exerted on the wall is not borne by the elastic zone of the latter. Cook's hypothesis is thus at least audacious and one must make sure, that it does not lead to contradictions. The inner radius of the wall is indicated by r_1 the outer radius by kr_1, the radius of the plastic front, by mr_1 and any variable radius, by lr_1 (see fig. 7).

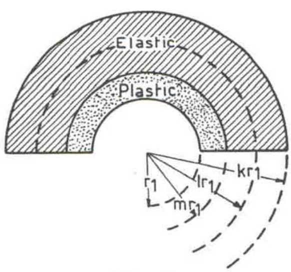


Fig. 7.

The zone, which remains elastic, will be first dealt with. Its diameter ratio is $kr_1/mr_1 = k/m$ the variable radius is in this region comprised between mr_1 and kr_1. The variations of the quantity l are thus comprised between m and k. As pressure p_m at $l = m$ reaches its yield value this value may be extracted from eq. (15), provided that p_{1y} is replaced by p_m and k by k/m

$$p_m = \left(1 - \frac{m^2}{k^2}\right) \tau_y. \tag{15}$$

Eqs. (9) to (13) are now applicable to the elastic zone by putting down $p_2 = 0$ and by replacing p_1 by above-mentioned p_m value, k by k/m and l by l/m. One forms thus eqs. (16) to (20) of table 2.

TABLE 2

Elastic zone ($m \leqslant l \leqslant k$)

$$\sigma_r = m^2 \left(\frac{1}{k^2} - \frac{1}{l^2} \right) \tau_y \tag{16}$$

$$\sigma_t = m^2 \left(\frac{1}{k^2} + \frac{1}{l^2} \right) \tau_y \tag{17}$$

$$\tau = \frac{m^2}{l^2} \tau_y \tag{18}$$

$$\sigma_z = \frac{m^2}{k^2} \tau_y \tag{19}$$

$$Eu = \left(\frac{1 - 2\nu}{k^2} + \frac{1 + \nu}{l^2} \right) m^2 \, l r_1 \, \tau_y \tag{20}$$

Plastic zone ($1 \leqslant l \leqslant m$)

$$\sigma_r = \left(\frac{m^2}{k^2} - 1 \right) \tau_y + \left(\log \frac{l^2}{m^2} \right) \tau_y' \tag{21}$$

$$\sigma_t = \left(\frac{m^2}{k^2} - 1 \right) \tau_y + \left(2 + \log \frac{l^2}{m^2} \right) \tau_y' \tag{22}$$

$$\tau = \tau_y' \tag{23}$$

$$\sigma_z = \left(\frac{m^2}{k^2} - 1 \right) \tau_y + \left(1 + \log \frac{l^2}{m^2} \right) \tau_y' \tag{24}$$

The plastic zone has a diameter ratio equal to $mr_1/r_1 = m$. The variable radius of this region is comprised between r_1 and mr_1, so that l varies between l and m. All that we know about the stresses in this zone is that $\sigma_t - \sigma_r = 2\tau$ and $\sigma_t + \sigma_r = 2\sigma_z$. This is however a valuable information, because following equalities may be written : $\sigma_t = \sigma_z + \tau$ and $\sigma_r = \sigma_z - \tau$ and because the stress configuration appears to be a combination of hydrostatic stress σ_z with a pure shear stress. Under these circumstances, we know from Crossland's experiments, that the plastic yield is not modified by σ_z and occurs as in a torsion test-pieces. By only considering a material, which in the course of a torsion test shows an upper yield shear stress τ_y and a lower yield shear stress τ'_y, giving rise to a yield without hardening, following equation $\sigma_t - \sigma_r = 2\tau'_y$ may be written and will be applicable everywhere in the plastic zone, provided that the overstraining is not too excessive which occurs, when k is not too great. With these restrictions, the equilibrium of the forces obeys eq. (2), which is applicable in the elastic as well as in the plastic zone and takes actually following form

$$\sigma_t - \sigma_r = 2\,\tau'_y = l\,\frac{d\sigma_r}{dl} \quad \text{or} \quad \int_l^m d\sigma_r = \tau'_y \int_l^m d \log l^2.$$

The right hand side of the integrated equation is equal to $\tau'_y \log (m^2/l^2)$; the left hand side of said equation is equal to $(\sigma_{r=mr_1} - \sigma_r)$; $\sigma_{r=mr_1}$ can be extracted from eq. (16) by equating l to m, because the stress σ_r is certainly continous throughout the wall; eq. (21) of table 2 is based on these indications; following equations of table 2 are found as follows : eq. (22) derives from eq. (21) and following relation : $\sigma_t = \sigma_r + 2\,\tau'_y$; eq. (24) derives from eqs. (21) and (22) and from following equation :
$\sigma_z = \frac{1}{2}(\sigma_r + \sigma_t)$ eq. (23) is mentioned as a reminder. It will be noted, that the radial displacement u in the plastic zone is not mentioned in table 2. This is accounted for by the fact, that the relation between the stresses and the resulting plastic strains depends upon the loading history.

It must be now ascertained, that the axial equilibrium of the forces is satisfied by Cook's assumption by means of which σ_z can be calculated in the elastic as well as in the plastic zone. This equilibrium is expressed by following relation

$$2\pi \int_1^m \sigma_z\,(\text{plast})\,(lr_1)\,d\,(lr_1) + 2\pi \int_m^k \sigma_z\,(\text{elast})\,(lr_1)\,d(lr_1) = \pi r_1^2\,p_1$$

or simplier by following relation

$$\int_1^m \sigma_z\,(\text{plast})\,dl^2 + \int_m^k \sigma_z\,(\text{elast})\,dl^2 = p_1.$$

By making use of eqs. (19) and (24), one can verify, that the left hand side of the above equation reduces to : $(1 - m^2/k^2)\,\tau_y + (\log m^2)\,\tau'_y$, which is precisely the value of the internal pressure, which value is obtained by putting down in eq. (21) $\sigma_{r=r_1} = -\,p_1$ and $l = 1$.

Cook's hypothesis is thus acceptable. One can even say, that this theory sufficiently explains the facts, if we consider, that it is a simplified one. These views are justified by theoretical and experimental considerations. COOK [1934] and CROSSLAND, JÖRGENSEN and BONES [1958] made use of Cook's hypothesis for interpreting their experiments and ALLEN and SOPWITH [1951] have demonstrated, that this hypothesis fits in well with their more elaborated theoretical solution. Table 2 improves a theory of the "autofrettage" expounded by DEFFET and GELBGRAS [1953]. It is also to be noted, that table 2 is applicable to a Maraging steel, when τ_y' is made equal to τ_y.

An m value being selected, one can find by means of the equations of table 2, how stresses are distributed in the elastic and plastic zones, when l varies. Then, by changing the m value, one can see how this stress distribution modifies, until the overstraining is completed ($m = k$). As the outer radius displacement u_2 is a dependent variable of the pressure, its variation can be found as follows : the displacement is extracted from eq. (20), where one puts down $l = k$ and pressure p_1 is extracted from eq. (21) where on puts down $\sigma_r = - p_1$ and $l = 1$; as m is a quantity, which may vary from $m = l$ to $m = k$, a diagram can be plotted by means of the corresponding u_2 and p_1 values. CROSSLAND, JÖRGENSEN and BONES [1958] used such a diagram as a reference curve, when they analyzed the results of tests, they carried out on a low-alloy steel. The results and the curve, show a very satisfactory concordance, although the dispersion of τ_y and τ_y' were not negligible.

The same authors have calculated the pressure p_{1y} by taking eq. (15) into consideration and using a corrected value of τ_y they have also calculated the "collapse" pressure p_{1c} after eq. (22) by putting down : $p_{1c} = -\sigma_r$, $m = k$ and $l = 1$

$$p_{1c} = \tau_y' \log k^2. \tag{25}$$

The results they have obtained by testing cylinders, made of a 0.15% carbon steel and of which the k ratio varied between $k = 1.5$ and $k = 6$ are in excellent agreement with eq. (25).

Although the simplified theory has been largely confirmed by tests, carried out to this purpose, such a theory cannot be applied to self-hooping a cylinder, without taking some precautions. In fact, when the pressure goes down, the cylinder shows residual stresses. The normal direction of the shear stress in the plastic zone is reversed and it may be feared, that as a consequence of the Bauschinger effect, a plastic deformation in the opposite direction prematurely appears. This effect can be detected on the hysteresis loop of the pressure vs. deformation curve pertaining to a cylinder submitted to pressures going up and down. When a cylinder at rest shows a plastic reversed deformation and when the pressure, to which it is submitted, goes up and down, its dimensions become unstable and its seals untight, so that it is finally destroyed. MACRAE [1930] has demonstrated, that a cylinder can be stabilized by submitting it to a progressive "autofrettage" combined with restoring heat treatments.

The simplified theory, which may be applied, as far as small deformations and a yield without hardening are concerned, has no far reaching consequences. It does not say anything about the ultimate pressure p_{1u}, to which a cylinder is submitted before bursting. MANNING [1945 and 1957] has elaborated a theory, which is so complete, that it is not only capable of correctly predetermining the p_{1u} value but also of following the evolution of the stresses developed in the wall from the very small strains up to greatest ones. Manning's theory is based on 3 hypotheses :

(a) There is no scale effect; stresses and strains only depend upon the k ratio.

(b) There is no axial strain, or better, this strain is negligible when it is compared with the others.

(c) The relation between the shear stress and the shear strain in the deformed cylindrical wall is the same as in a torsion test rod.

Hypothesis (a) entitles us to consider a cylinder, of which the inside radius is equal to 1 and the outside radius, to k. Hypothesis (b) combined with the fact, experimentally well established, that the density of the material remains constant, when the material yields, amounts to writing following relation

$$\pi (r + u)^2 - \pi (1 + u_1)^2 = \pi r^2 - \pi \quad \text{or} \quad u (2r + u) = u_1 (2 + u_1). \tag{26}$$

This equation expresses the fact, that the mass comprised between both circles is the same before and after deformation, the inner circle selected being this one of the bore. Eq. (2), adapted to great radial displacements and in which $\sigma_t - \sigma_r$ has been replaced by 2τ may be written as follows

$$2\tau = (r + u) \frac{d\sigma_r}{d(r + u)} \quad \text{or} \quad - \int dp = 2 \int \frac{\tau}{r + u} d(r + u). \tag{27}$$

The integrals are to be calculated between limits, which will be determined later on. Table 3 can be now completed and its first column contains all the radii, which are to be taken into account. The second column contains the displacements of these radii and begins with u_1, the value of which is arbitrarily chosen equal to the unit of length; following values have been calculated by making use of eq. (26). The third column contains the deformed radii and the forth column the ratios u/r.

We could define a shear strain, as MANNING [1945] did, and utilize the classical values $\varepsilon_t = u/r$ and ε_r. But it is perhaps preferable to improve

TABLE 3

	u/r	$2 \log \left(1 + \frac{u}{r}\right) = \gamma$	τ
0	1.0000	1.3863	12.5
7	0.8652	1.2469	12.5
[illegible]	0.7759	1.1280	12.3
5	0.6659	1.0207	12.1
[illegible]	0.5908	0.9285	11.8
3	0.5274	0.8471	11.5

the notion of ε_t as follows; we consider the moment when radius r has become equal to s ($s < u$); at the moment when s has become $s + \delta s$, the length of the circumference, of which the radius is equal to s, is strained by $\delta \varepsilon_t = (\delta s)/s = \delta \log s$. The density being constant, one can write : $(1 + \delta \varepsilon_t)(1 + \delta \varepsilon_r) = 1$ that is to say : $\delta \varepsilon_r = - \delta \varepsilon_t$. On the other side, the shear strain δ_y may be defined by the egality : $\delta_y = \delta(\varepsilon_t - \varepsilon_r) = 2 \delta \log s$, which may be integrated as follows

$$\gamma = 2\int_r^{r+u} \delta \log s = 2 \log \left[1 + \frac{u}{r}\right].$$

The fifth column of table 3 is this one of the so-called "natural shear strains". The shear strains in the cylindrical wall are then compared with torsion test shear strains. By making use of Manning's hypothesis (c), it becomes then possible, to fill the sixth column of table 3 as well as the following column. In fact the figures put in the sixth column are fictitious ones but their presence is necessary to get a clear insight into the matter.

Eq. (27) shows, that following relation is absolutely true

$$p_{2.2271} - p_{2.2913} = 2\int_{2.2271}^{2.2913} \frac{\tau}{r + u}\, \mathrm{d}\,(r + u)$$

$$= 2\, \overline{\left(\frac{\tau}{r + u}\right)}\, (2.2913 - 2.2271).$$

If we assume, that the true mean of $\tau/(r + u)$ is equal to its arithmetical mean $\frac{1}{2}$ (5.02 + 5.30) following relation is then only approximately true

$$p_{2.2271} = p_{2.2913} + (5.02 + 5.30)\,(2.2913 - 2.2271) = p_{2.2913} + 0.662$$

Let us now consider the case of a cylindrical wall, of which the initial radii are respectively 1 and 15 and of which the inner radius has undergone the strong displacement $u_1 = 1$, because the inner surface of the cylindrical wall has been submitted to an internal pressure p_1 still unknown. One puts down $p_{2.2913} = 0$ and thus $p_{2.2271} = 0.662$. One can write

$$p_{2.1656} = p_{2.2271} + (5.30 + 5.59)\,(2.2271 - 2.1656)$$
$$= p_{2.2271} + 0.670 = 0.662 + 0.670$$

and so on. Figures 0.662, 0.670 etc... are put down in the eighth column of table 3, from the bottom upwards. The last column contains the cumulated sums of these figures, put down from the bottom upwards. This last column shows us, which are the pressures existing in the wall and particularly shows, that the internal pressure is equal to 3.30. Stress σ_t, is calculated by making use of following relation $\sigma_t = 2\tau + \sigma_r = 2\tau - p$. The problem is thus solved, provided that stress σ_z is overlooked.

In the earlier version of his method, MANNING [1945] has shown on a concrete example, how it is possible to predetermine the ultimate pressure p_{1u}, a cylinder is submitted to, before bursting. By considering increasing displacements u_1 of the inner radius and by computing for each of the displacements the internal pressure by means of a table, it has been found, that from a certain value of u_1, onwards, pressure p_1 decreases, passing thus through a maximum, which is identified with p_{1u}. This is obviously a tedious and time consuming method.

The improved version [1957] of this method is based on table 3, extended up to $r_2 = 28$ for instance, which is indeed a very high value of the k ratio. The external pressure corresponding to $r_2 = 28$ being assumed to be equal to zero, the other pressures are calculated by making summations up to the inner radius. At a certain place, the pressure will be found equal to τ_y (1-1/28^2) which is the critical yield pressure according to eq. (15). Such a place is thus the boundary between the elastic zone and the plastic one. An example of such a table in which radius r takes all the values between 1 and 28 has been given by CROSSLAND, JÖRGENSEN and BONES [1958]. A diagram has been plotted by utilizing the data of this table and a curve obtained, which is shown qualitatively on fig. 8 and for convenience sake a logarithmic scale has been used for representing the abcissae.

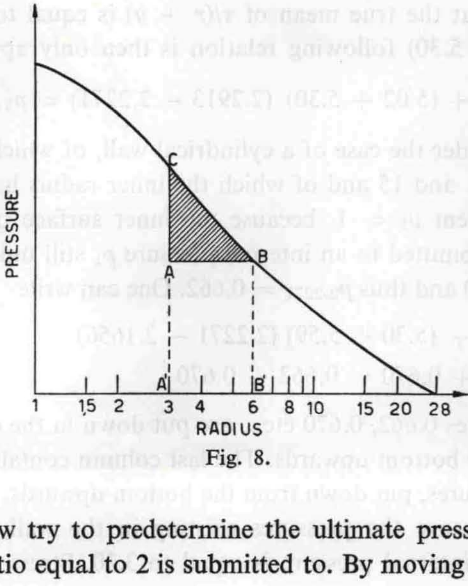


Fig. 8.

We will now try to predetermine the ultimate pressure p_{1u}, which a wall with a k ratio equal to 2 is submitted to. By moving from right to left along the pressure curve, the right angle ABC, of which the side AB is equal to $^{10}\log 2$ in length, the curve intercepts on the vertical side of the angle a variable length AC, which admits of a maximum equal to p_{1u}, because the curve is so shaped. This way of looking at things supposes, that the wall in the situation ABC of the figure, that is to say, submitted to an internal pressure $A'C$ and an external pressure $B'B$ yields as a wall submitted to the sole internal pressure AC, combined with a purely hydrostatic state $B'B$, which is legitimate. It is then obvious, that the displacement ratios u/r must be corrected by subtracting from their values the pure elastic contribution, attributable to the hydrostatic state $B'B$.

Manning's theory has been tested by CROSSLAND, JÖRGENSEN and BONES [1958]; as far as great deformations and pressures p_{1u} are concerned, theory and practice give concordant results as well for a 0.15% carbon steel as for a low-alloy steel. By the way let us say, that the correct value of p_{1u} can only be measured, when the appropriate test is very slowly carried out. Otherwise the cylinder would appear to be too resistant owing to the fact, that the wall had not time enough for thinning down, as much as required. It is true, that FAUPEL [1956] has mentioned grievous divergences between his results and Manning's predeterminations but as he does not say anything about the torsion test rods and the former utilization of the cylinders, it is most likely, that the test rods steel was not in the same state as the cylinders steel.

Manning's theory has the same disadvantage as the elementary theory and eq. (15). Torsion tests results are necessary for applying this theory. One has thus tried to compute p_{1u} by means of empirical formulae such as the two following ones, which have been taken into consideration by CROSSLAND and BONES [1958] and criticized by the same :
FAUBEL and FURBECK [1953] formula :

$$p_{1u} = \frac{2\sigma_y'}{\sqrt{3}} \left[2 - \frac{\sigma_y'}{\sigma_u}\right] \log k,$$

average diameter formula :

$$p_{1u} = 2\sigma_u \, \frac{k-1}{k+1},$$

in which σ_y is the upper yield stress in tension, σ_y', the lower yield stress and σ_u the ultimate tensile stress.

Unfortunately, GLADKOVSKII, VERESHCHAGIN and IVANOV [1958] have mentioned that their results do not agree with both above-mentioned empirical formulae neither with others previously mentioned by CROSSLAND and

BONES [1955]. On the other side, LIALINE [1959] has demonstrated, that by slightly modifying the average diameter formula

$$p_{1u} = (\sigma_y + \sigma_u)\frac{k-1}{k+1}$$

the question of safety is duly taken into account, provided that one utilizes the 0.01% conventional yield point σ_y, as far as steel grades are concerned of which the upper yield stress is not accurately defined.

An empirical formula, which has been proposed by LEINS [1955] and has been carefully studied by DAVID [1956] is worth mentioning

$$(k-1)\frac{\sigma_u}{p_{1u}} = \alpha + \beta(k-1)$$

Unfortunately this formula cannot be immediately applied. The parameters α and β, which are typical of the selected steel, can only be determined by carrying out bursting tests on cylinders.

6. The Shape, the Structure and the Chemical Attack

Their effects

The attention of the reader will here be drawn to certain effects which often affect pressure vessels and may be dangerous. The effects of the temperature will be separately grouped in the following section for convenience sake, owing to the fact, that some of the phenomena here dealt with, such as corrosion and ageing definitely depend upon the temperature.

When we expounded the theory of the thick-walled cylinder, we only considered the smooth part of the wall, the ends of which we systematically disregarded. Now it is precisely at the ends that shape irregularities are to be found : more or less sudden modifications of the diameter, threads, holes etc. Such irregularities modify the stress field and cause the material to yield locally and precociously. This yielding is not to be considered as a phenomenon capable of doing a lot of harm. On the contrary, it is rather a phenomenon, which it can be taken advantage of, provided it can normally develop. All that hinders a material from easily yielding must consequently be avoided such as the re-entrant corners for instance or especially a material insufficiently ductile because it has been submitted to too severe a hardening.

A material submitted to variable stresses becomes "tired". It is therefore not necessary that the algebraic sign of the stress change, as it is for instance the case, when reversed bending tests are carried out. It is sufficient that a certain stress inversion take place as a stress decrease succeeding a stress increase. Thus the maximum shear stress in an elastic cylinder simply comes back round zero, when the internal pressure is released. The fractures caused by the metal fatigue suddenly occur without being announced by a suspicious deformation of the material. By their appearance by which specialists are not taken in, one recognizes a destruction caused by a crystalline decohesion. One would not easily understand, why a metal perfectly sound and elastically acted upon, breaks, because it is "tired", if it could not be reasonably assumed, that it is the seat of a microplasticity phenomenon, which is hardly perceptible for it is probably in these tiny regions, that the destruction occurs, such destruction being accounted for by crystalline lamellae moving in opposite directions and rubbing one against the other.

The parts of a compressor, submitted to numerous and rapid pulsations of the pressure are necessarily to be counted among those of which the fatigue has to be taken into account, when their dimensions are to be calculated. But the apparatuses, which slowly but continuously work such as the mercury pumps are also themselves threatened by the fatigue. Such fractures have already been reported. Reversed tensile and bending fatigue tests are abundantly dealt with in the technical literature. It seems however, that the solution of our problem can hardly be based on their results. Torsional tests and tests carried out on hollow cylinders submitted to alternating pressures were found more suitable to the purpose by MORRISON, CROSSLAND and PARRY who carried out such tests [1956, 1957, 1959, 1960a and 1960b]. It was also found that for each steel there is an upper limit of the shear stress τ below which the test piece is not yet broken after 10 millions of cycles, whereas above this limit of τ, the test piece breaks the quicker as the value of τ is higher. It is obvious that a design is to be based on this limit. A lot of experiments have been carried out but their number appears to be not yet high enough for determining with great precision the fatigue limit and expressing it by a percentage of the tensile ultimate stress. As far as a ferritic or martensitic steel is concerned, it seems not unreasonable to say that such a percentage may amount to 30%.

Different methods have been tried with a view to increasing the fatigue endurance. The bore of the cylinder has been nitrided. As the nitride layer's volume slightly increases in the course of this treatment, this layer is compressed by the remainder of the wall and that is a favourable circumstance because the small defects occuring in the immediate vicinity of the bore less increase the value of the stresses when the cylinder is submitted to a pressure. An other method which seems to be an adequate one, consists in reinforcing the cylinder by "autofrettage" with a view to putting it under the same favourable circumstances. As the self-reinforced cylinder is obviously the seat of a phenomenon called "hysteresis" and wastes energy, one could believe that the fatigue endurance is diminished. In fact it is the contrary which occurs. A last method, which is the simpliest one consists in choosing a material as homogenous as possible, such as a steel obtained by consumable electrode remelting.

The ageing is also a phenomenon which is connected to the structure of the material as the fatigue is. We will not dwell upon this phenomenon, which is too well known but one will remember that a steel runs the risk of becoming brittle by ageing.

Corrosion by chemical agents is a term which covers numerous specific cases. For instance, it is obvious that a steel decarburized by hydrogen under pressure is entirely different from a steel attacked by an acid and showing corrosion pits. As a rule one recommends to choose the metal which best resists the chemical agent considered and to carefully polish the interior of the vessel. If the mechanical properties of the metal are too weak, the cylinder can be reinforced or self-reinforced or any method of the same kind can be applied to the same purpose.

7. The Effects of the Temperature

The unpleasant effects of the temperature upon the steels and alloys are mainly the following ones :

a) appearance of temperature stresses,

b) creep at high temperatures, impairing the strength of the material,

c) possible embrittlement of the material at low temperatures. Let us now briefly discuss these three items.

a) Stresses resulting from an unequal expansion of the material appear in a cylindrical wall as soon a temperature gradient appears. These stresses are the temperature ones, which may become dangerous, when they are superimposed upon the pressure ones. When one assumes — and that is a very restrictive assumption indeed — that the distribution of the temperatures obeys the law of the cylindrical symmetry and varies neither in the axial direction nor throughout the time, that the axial strain is constant and that the Young modulus as well as Poisson's ratio and other characteristics of the material are constant within the interval of temperature considered the stresses can be very easily expressed in a mathematical form. In less idealized cases, which most frequently happen in practice the equations get more complicated or stay roughly approximative. For instance, a superheater tube of which a side is exposed to furnace radiations is the seat of unsymmetrical axial stresses, which incurvate the axis of the tube. An other example which gives rise to a lot of theoritical difficulties is the thermal shock effect. Some other unpleasant things may sometimes happen such as jamming, cold tightenings becoming loose or simply the difficulty of finding a suitable, heat resisting packing.

b) When its temperature rises, a vessel gradually loses its elastic properties and begins to creep. The duration of its life becomes necessarily limited. When the pressure to which it is submitted, is kept constant,this duration is the shorter as the temperature reaches a higher level. One would be wrong to believe that a material only creeps at a furnace temperature. Lead already creeps at the room temperature. In fact, the temperature at which the material begins to creep, is "grosso modo" equal to half the temperature corresponding to the melting point of the material, the temperatures being read on the Kelvin scale. Such a temperature is equal to 20°C for the lead and approximately 600°C for the alloys with high nickel content. For the time being, among the nickel alloys, it is the "nimonic" which best withstands the temperature but cannot be easily machined at room temperature. A very fine colloïdal dispersion of refractory oxides is actually made use of, by metallurgists with a view to improve the high temperature performance of the metal in which the oxides are incorporated. This process is applicable to numerous metals (copper, aluminium, cobalt, iron, wolfram, molybdenum, nickel and nickel-chromium) to which sundry dispersion agents are incorporated, among which we mention the thorium and aluminium oxides, and the titanium and lanthanium ones. The sole metal, which is so consolidated and sold on the market is the "T.D. Nickel" (98% nickel and 2% thorium oxide). Its tensile strength is equal to 7 kg/mm; at 1 315°C, the nickel melting point being 1 453°C. As the thorium oxide is insoluble, the "T.D. Nickel" keeps its strength at very high temperatures, whereas the more classical superalloys lose it because the hardening agent dissolves in them. The "T.D. Nickel" is a very good conductor of heat and electricity. It withstands the fatigue, the corrosion the oxidation at high temperatures and can be easily machined.

Let us now mention by the way the "Maraging steel" of which the mechanical properties and the ductility at the room temperature are absolutely noteworthy. As soon as this steel will come into general use, the efficiency of the classical pressure apparatuses will be increased in the range of the elastic strain and perhaps will it be attractive to overstrain such apparatuses, so that the attention of our contemporaries would again be drawn to the problem of the autofrettage. These iron-nickel alloys with a nickel content amounting to 18-30% have a martensitic structure. The hot ageing of this martensite accounts for the mechanical properties of this nickel steel and justifies its commercial name.

As far as we know, there is actually no theory which can be relied on and is capable of estimating how long a thick-walled cylinder creeping under pressure at a high temperature can live. The theory about creep expounded by MANNING [1957] deserves to be mentioned to the reader, although the opinions expressed about it are divided.

c) In general the elastic limit of a steel, its ultimate strength its fatigue endurance increase at low temperatures. However if the yield strength draws too much nearer to the tensile strength, the steel becomes dangerously brittle. All things happen as if we had to do at the room temperature, with a steel which would have been too much hardened and it has been explained in the preceding section, why a vessel cannot be excessively hardened. Fortunately some rustless austenitic steel grades do not become brittle at low temperatures. It is a long time since we know that a good correlation exists between the crystalline structure of a material, its brittleness or its ductility at low temperature. Metals with body centered cubic lattice become brittle, whereas those with face-centered cubic lattice remain sufficiently ductile. The question of how the different steel grades behave at low temperatures has been very clearly dealt with by SCOTT [1958] in his treatise in which the reader will find references to a more specialized literature.

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